Technical University of Cluj-Napoca, Romania
Faculty of Automation and Computer Science
Department of Mathematics

"Seminarul Theodor Angheluță" Association

Conference Book

61st International Symposium on Functional Equations

Dedicated to the memory of Jürg Rätz

June 15–21, 2025
Cluj-Napoca, Romania

https://isfe.uken.krakow.pl/61/

Welcome to Cluj-Napoca

Dear Participants,

We are delighted to hold the *61st International Symposium on Functional Equations* this year, under the auspices of the **Department of Mathematics of the Technical University of Cluj-Napoca** and **“Seminarul Theodor Angheluță” Association**, and we are honored to have you as our guests.

Located in the heart of Transylvania, **Cluj-Napoca** is one of Romania’s most vibrant and historic cities. Known for its rich cultural heritage, dynamic academic environment, and youthful energy, Cluj-Napoca is a hub of innovation and intellectual exchange. With its blend of medieval charm and modern development, the city offers a unique atmosphere, where centuries-old churches and fortifications stand alongside lively cafés, art galleries, and cutting-edge research institutions. As the home of several prestigious universities, including the **Technical University of Cluj-Napoca**, the city plays a central role in shaping Romania’s academic and scientific landscape. We hope you will find time to explore its streets, enjoy its hospitality, and experience the warmth and creativity that define Cluj-Napoca.

This symposium has long served as a unique platform for the international mathematical community to come together and explore the fascinating world of functional equations, celebrating both its classical foundations and modern developments. We hope that the 61st edition of the ISFE continues to inspire dialogue, discovery, and collaboration across generations and disciplines.

The Organizing Committee

Scientific Commitee

Ordinary Members

Gian Luigi Forti (University of Milan, Italy)
Roman Ger (Silesian University, Poland)
Zsolt Páles (University of Debrecen, Hungary)
Maciej Sablik (Silesian University, Poland) – chairman
Jens Schwaiger (University of Graz, Austria)
Ekaterina Shulman (Silesian University, Poland)
László Székelyhidi (University of Debrecen, Hungary)

Honorary Members

Jürg Rätz (University of Bern, Switzerland)
Ludwig Reich (University of Graz, Austria)

61st International Symposium on Functional Equations

Organizing Committee

Dorian Popa (Technical University of Cluj-Napoca) – chairman
Ioan Rașa (Technical University of Cluj-Napoca) – co-chairman
Alina Ramona Baias (Technical University of Cluj-Napoca) – scientific secretary
Diana Otrocol (Technical University of Cluj-Napoca) – scientific secretary
Mircea Rus (Technical University of Cluj-Napoca) – technical secretary
Daniela Inoan (Technical University of Cluj-Napoca)
Daniela Marian (Technical University of Cluj-Napoca)
Liana Timboș (Technical University of Cluj-Napoca)
Delia Kerekes (Technical University of Cluj-Napoca)
Paweł Solarz (University of the National Education Commission, Kraków) – web administrator

Dear Participants,

Welcome to the "Grand Hotel Napoca" in Cluj-Napoca.

Hotel

The "Grand Hotel Napoca" is a 4-star hotel located in the heart of Cluj-Napoca, on Octavian Goga Street, No. 1, right next to the Central Park and the "Someșul Mic" river. Its excellent location puts it only a few minutes' walk from the Cluj Arena, the Old Town, and other local attractions. "Avram Iancu" Cluj International Airport is about a 20-minute drive from the hotel, while Cluj-Napoca Central Train Station is a 15-minute walk away.

Talks

The duration of a regular talk should not exceed 20 minutes, with an additional 5 minutes allocated for discussion. If a speaker finishes earlier, the next talk will start immediately.

Meals and Drinks

The meals will be served in a self-service system, with vegetarian options included. Red and white wine will be served at dinner. You can order other drinks from the bar at your own expense. The first meal is dinner on Sunday, June 15 – it will be served from 7 PM to 9 PM. The last meal is breakfast on Saturday, June 21.

Coffee Breaks

There will be two coffee breaks per day, one in the morning and one in the afternoon.

Excursion

The excursion will take place on Wednesday, June 18, after lunch. We will go to the Salt Mine in Turda – the visiting of the mine will take approximately 2 hours. Warm clothing is recommended inside the salt mine due to the low temperature ($10° - 12°$ C). Dinner will be served at the restaurant "Sarea în Bucate", located close to the salt mine, where there will also be a wine tasting.

Festive Dinner and Closing Ceremony

The festive dinner will take place on Thursday, June 19. The closing ceremony will be held on the morning of Saturday, June 21, after breakfast.

For an up-to-date version of the conference program, please visit the official website at:

`https://isfe.uken.krakow.pl/61/index.php?id=10`

SCAN ME

BAIAS, Alina Ramona
Technical University of Cluj Napoca, Romania
@ baias.alina@math.utcluj.ro

BESSENYEI, Mihály
University of Miskolc, Hungary
@ mihaly.bessenyei@uni-miskolc.hu

BOROS, Zoltán
University of Debrecen, Hungary
@ zboros@science.unideb.hu

CĂDARIU, Liviu
Politehnica University of Timișoara, Romania
@ liviu.cadariu-brailoiu@upt.ro

CHMIELIŃSKI, Jacek
University of the National Education Commission, Krakow, Poland
@ jacek.chmielinski@uken.krakow.pl

FÖRG-ROB, Wolfgang
University of Innsbruck, Austria
@ wolfgang.foerg-rob@uibk.ac.at

FORTI, Gian Luigi
Università degli Studi di Milano, Italy
@ gianluigi.forti@unimi.it

GER, Roman
Silesian University, Katowice, Poland
@ romanger@us.edu.pl

GILÁNYI, Attila
University of Debrecen, Hungary
@ gilanyi@inf.unideb.hu

GŁAZOWSKA, Dorota
University of Zielona Góra, Poland
@ d.glazowska@im.uz.zgora.pl

GOPALAKRISHNA, Chaitanya
Indian Statistical Institute, Bangalore, India
@ cberbalaje@gmail.com

GRÜNWALD, Richárd
🏛 University of Debrecen, Hungary
@ richard.grunwald@science.unideb.hu

GSELMANN, Eszter
🏛 University of Debrecen, Hungary
@ gselmann@science.unideb.hu

HORVÁTH, László
🏛 Department of Mathematics, University of Pannonia, Hungary
@ horvath.laszlo@mik.uni-pannon.hu

INOAN, Daniela
🏛 Technical University of Cluj-Napoca, Romania
@ Daniela.Inoan@math.utcluj.ro

IQBAL, Mehak
🏛 University of Debrecen, Hungary
@ iqbal.mehak@science.unideb.hu

JARCZYK, Justyna
🏛 Institute of Mathematics, University of Zielona Góra, Poland
@ j.jarczyk@im.uz.zgora.pl

JARCZYK, Witold
🏛 Institute of Mathematics, University of Zielona Góra, Poland
@ w.jarczyk@im.uz.zgora.pl

KEREKES, Delia-Maria
🏛 Technical University of Cluj-Napoca, Romania
@ Delia.Kerekes@math.utcluj.ro

KISS, Tibor
🏛 University of Debrecen, Hungary
@ kiss.tibor@science.unideb.hu

KLARIČIĆ BAKULA, Milica
🏛 University of Split, Faculty of Science, Croatia
@ milica@pmfst.hr

LEŚNIAK, Zbigniew
🏛 Department of Mathematics, University of the National Education Commission, Krakow, Poland
@ zbigniew.lesniak@uken.krakow.pl

MĂDUȚA, Alexandra
🏛 Technical University of Cluj-Napoca, Department of Mathematics, Romania
@ Alexandra.Maduta@math.utcluj.ro

MARIAN, **Daniela**
🏛 Technical University of Cluj-Napoca, Department of Mathematics, Romania
@ Daniela.Marian@math.utcluj.ro

MOTRONEA, **Gabriela**
🏛 Technical University of Cluj-Napoca, Romania
@ gdenisa19@gmail.com

NICULESCU, **Constantin P.**
🏛 University of Craiova, Romania
@ constantin.p.niculescu@gmail.com

NOVAC, **Adela**
🏛 Technical University of Cluj-Napoca, Romania
@ adela.novac@math.utcluj.ro

OKAMURA, **Kazuki**
🏛 Shizuoka University, Japan
@ okamura.kazuki@shizuoka.ac.jp

OTROCOL, **Diana**
🏛 Technical University of Cluj-Napoca, Department of Mathematics, Romania
@ diana.otrocol@math.utcluj.ro

PAICU, **Alexandra**
🏛 Technical University of Cluj-Napoca, Romania
@ paicu_alexandra98@yahoo.com

PÁLES, **Zsolt**
🏛 Institute of Mathematics, University of Debrecen, Hungary
@ pales@science.unideb.hu

PASTECZKA, **Paweł**
🏛 University of the National Education Commission, Krakow, Poland
@ pawel.pasteczka@uken.krakow.pl

POPA, **Dorian**
🏛 Technical University of Cluj-Napoca, Romania
@ popa.dorian@math.utcluj.ro

RAȘA, **Ioan**
🏛 Technical University of Cluj-Napoca, Romania
@ ioan.rasa@math.utcluj.ro

REICH, **Ludwig**
🏛 University of Graz, Austria
@ ludwig.reich@uni-graz.at

RELA, **Patryk**
🏛 University of Rzeszów, Poland
@ `prela@ur.edu.pl`

RUS, **Mircea Dan**
🏛 Technical University of Cluj-Napoca, Romania
@ `rus.mircea@math.utcluj.ro`

SABLIK, **Maciej**
🏛 University of Silesia, Poland
@ `maciej.sablik@us.edu.pl`

SÎNGEORZAN, **Alexandra**
🏛 Technical University of Cluj-Napoca, Romania
@ `alexandra.singeorzan@yahoo.com`

SZÉKELYHIDI, **László**
🏛 University of Debrecen, Hungary
@ `lszekelyhidi@gmail.com`

SZOSTOK, **Tomasz**
🏛 University of Silesia, Poland
@ `tomasz.szostok@us.edu.pl`

TIMBOȘ, **Liana**
🏛 Technical University of Cluj-Napoca, Romania
@ `Liana.Timbos@math.utcluj.ro`

TO, **Lan Nhi**
🏛 University of Nyiregyhaza, Hungary
@ `lan.nhi.to@science.unideb.hu`

TÓTH, **Norbert**
🏛 Institute of Mathematics, Faculty of Science and Technology, University of Debrecen, Hungary
@ `toth.norbert@science.unideb.hu`

TÓTH, **Péter**
🏛 University of Debrecen, Hungary
@ `toth.peter@science.unideb.hu`

VERNESCU, **Andrei**
🏛 University of Târgoviște, Romania
@ `avernescu@gmail.com`

List of Talks

« Je n'ai fait celle-ci plus longue que parce que je n'ai pas eu le loisir de la faire plus courte. »
(*"I have made this longer than usual because I have not had time to make it shorter."*)

— **Blaise Pascal**

The best Ulam constant of a first-order linear differential operator

Alina Ramona Baias
Technical University of Cluj Napoca, Romania
(joint work with **Dorian Popa and Alexandra Paicu**)

The aim of this paper is to give an Ulam stability result for the linear differential operator $D: C^1(\mathbb{R}, X) \to C(\mathbb{R}, X)$ defined by $Dy = y' + f \cdot y$, where X is a Banach space over $\mathbb{C}$. Moreover, if there exists $\inf_{x\in\mathbb{R}} |\Re f(x)| = m$ and $m \neq 0$, we prove that $K = \frac{1}{m}$ is the best Ulam constant of the operator. As applications, we provide some stability results for Hill's operator and for the n-th order linear differential operator.

References:

[1] A.R. Baias, Popa, D, *On the best Ulam constant of the second order linear differential operator*, RACSAM 114, 23 (2020). https://doi.org/10.1007/s13398-019-00776-4

[2] A.R. Baias, Popa, D. *On the best Ulam constant of a higher order linear difference equation*, Bull. Sci. Math., , **166**, art no. 102928, https://doi.org/10.1016/j.bulsci.2020.102928, (2021).

[3] D. Popa, I. Rasa, *On the Hyers-Ulam stability of the linear differential equation*, J. Math. Anal. Appl., **381**, 530–537, (2011).

Toader convexity revisited

Mihály Bessenyei
University of Miskolc, Hungary
(joint work with **Dávid Kertész**)

The aim of the talk is to revisit the convexity notion introduced by Toader [2]. A part of the results are known [1]. However, our approach birngs new insight by replacing the topic into the context of segment convex structures [3]. The main results link classical and Toader convexity and enable us to use the standard tools of convex and combinatorial geometry directly. As applications, we improve some earlier results and present new ones, as well.

References:

[1] Lara, T., Merentes, N., Páles, Zs., Quintero, R., and Rosales, E.: *On m- convexity on real linear spaces*, UPI J. Math. Biostat. **1** (2018), JMB8.

[2] Toader, G. H.: *Some generalizations of the convexity*, Proc. Colloq. Approx. Optim. Cluj-Napoca (Romania) 1984, pp. 329–338.

[3] van de Vel, M. L. J.: *Theory of convex structures*, North-Holland Mathematical Library, vol. 50, North-Holland Publishing Co., Amsterdam, 1993.

◇ ◇ ◇

An alternative equation for polynomial functions on hyperbolas

Zoltán Boros
University of Debrecen, Hungary
(joint work with **Rayene Menzer**)

We consider generalized polynomials $f, g : \mathbb{R} \to \mathbb{R}$ that satisfy the additional equation $f(x)g(y) = 0$ for the pairs $(x, y) \in D$, where $D \subset \mathbb{R}^2$ is given by some algebraic or analytic condition. In particular, when a, $b \in \mathbb{R} \setminus \{0\}$ and $D = \{ (x, y) \in \mathbb{R}^2 \mid \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \}$, as well as in some further cases, we prove that f or g is identically equal to zero. An analogous result for ellipses has been published recently by Bruce Ebanks as well.

◇ ◇ ◇

The stability of a linear functional equations in a single variable using the fixed point method

Liviu Cădariu

Politehnica University of Timișoara, Romania

In this talk we prove that a result concerning the generalized Hyers-Ulam stability of the linear functional equation $g(\varphi(x)) = a(x) \bullet g(x)$ on a complete metric group can be obtained using the fixed point technique. Moreover, we give a characterization of the functions that can be approximated with a given error, by the solution of the linear equation mention above.

References:

[1] Cădariu, L.; Găvruța, L.; Găvruța, P. Fixed points and generalized Hyers-Ulam stability, *Abstr. Appl. Anal.* **2012** Article ID 712743, (2012) 10 pp.

[2] Cădariu, L.; Manolescu, L. Fixed points and the stability of the linear functional equations in a single variable. *Carpathian Journal of Mathematics* **38** (2022), no. 3, 769–776.

[3] Jung, S.- M.; Popa, D.; Rassias, M.T. On the stability of the linear functional equation in a single variable on complete metric groups. *Journal of Global Optimization* **59** (2014), no. 1, 165–171.

[4] Kim, G.H.; Rassias, Th.M. On the Stability of the Generalized Psi Functional Equation. *Axioms* **9** (2020), no. 2.

◇ ◇ ◇

Approximate orthogonality via norm derivatives in complex Banach spaces

Jacek Chmieliński

University of the National Education Commission, Krakow, Poland

(joint work with **Najla Altwaijry, Cristian Conde, Kais Feki**)

In a complex normed space X, for $x, y \in X$ and $\theta \in [0, 2\pi)$ we define

$$\rho'_\theta(x,y) = \lim_{r\to 0^+} \frac{\|x + re^{i\theta}y\|^2 - \|x\|^2}{2r} = \|x\| \lim_{r\to 0^+} \frac{\|x + re^{i\theta}y\| - \|x\|}{r},$$

extending the idea of classical norm derivatives $\rho'_\pm$. Next, we use this notion to obtain characterizations of the approximate Birkhoff-James orthogonality in general complex normed linear spaces, as well as in spaces of linear and bounded operators. The talk is based on a recent paper [1].

References:

[1] N. Altwaijry, J. Chmieliński, C. Conde, K. Feki, *Approximate orthogonality and its applications to specific classes of linear operators*, Bull. Sci. math. 202 (2025), 103646.

◇ ◇ ◇

Regular solutions of an alternative additive-quadratic equation

Gian Luigi Forti

Università degli Studi di Milano, Italy

The following alternative equation is investigated

$$f(x+y)-f(x)-f(y)\neq 0 \text{ implies } g(x+y)+g(x-y)-2g(x)-2g(y)=0,$$

where $f,g:\mathbb{R}\to\mathbb{R}$, f is in $C^1(\mathbb{R})$ and g in $C^2(\mathbb{R})$. The regularity conditions allow a useful integral representation of the Cauchy and quadratic difference of the two functions f and g, indeed we have

$$\begin{aligned} f(x) &= \int_0^x \phi(t)dt + E_1x + E_2, \quad E_2 = f(0),\\ g(x) &= \int_0^x \sigma(t)dt + \frac{C_1}{2}x^2 + C_2x + C_3, \quad C_3 = g(0), \end{aligned}$$

where ϕ is continuous and σ is in $C^1(\mathbb{R})$, with $\phi(0)=0=\sigma(0)=\sigma'(0)$. The alternative equation becomes

$$\Big[\int_0^y [\phi(x+t)-\phi(t)]dt - E_2)\Big]\times$$
$$\Big[\int_0^y [\sigma(x+t)+\sigma(x-t)-2\sigma(t)]dt - 2C_2y + 2C_3\Big] = 0.$$

It is proved that in these conditions the equation has only trivial solutions, that is either $f(x)=\alpha x$, $x\in\mathbb{R}$, or $g(x)=\beta x^2$, $x\in\mathbb{R}$, for some $\alpha,\beta\in\mathbb{R}$.

Some open problems are then presented.

References:

[1] G.-L. Forti, L. Paganoni: *On an alternative Cauchy equation in two unknown functions. Some classes of solutions*, Aequationes Math. **42** (1991), 271–295.

[2] G.-L. Forti: *A note on an alternative quadratic equation*, Annales Univ. Budapest., Sect. Comp. **40** (2013), 223–232.

[3] G.-L. Forti, B. Wilkens: *On an alternative additive-quadratic functional equation.* Aequationes Math. 99 (2025), 591-610.

[4] M. Kuczma: *Functional Equations on Restricted Domains.* Aequationes Math. **18**(1978), 1–34.

◇ ◇ ◇

The non-existence of a Hamel-basis for open cones in real normed linear spaces

Roman Ger

Silesian University, Katowice, Poland

János Aczél and Paul Erdős have shown in [1] that there does not exist a Hamel-basis of the nonnegative numbers the elements of which were nonnegative real numbers and any nonnegative real number could be represented in a unique way as linear combinations of the basis elements with nonnegative rational coefficients. What about the first quadrant in $\mathbb{R}^2$? The answer is negative as well. More generally, we show that any open cone in a real normed linear space contains no Hamel-basis.

References:

[1] J. Aczél & P. Erdős, *The non-existence of a Hamel-basis and the general solution of Cauchy's functional equation for nonnegative numbers*, Publicationes Mathematicae Debrecen **12** (1965), 259-263

◇ ◇ ◇

Bernstein–Doetsch and Sierpiński theorems for (M, N)-convex functions

Attila Gilányi

University of Debrecen, Hungary

(joint work with **Zsolt Páles**)

One of the fundamental results of the classical theory of convex functions is the Bernstein–Doetsch theorem ([1]), which states that if a real-valued Jensen-convex function defined on an open interval is locally bounded above at a point in its domain, then it is continuous. According to a related result by W. Sierpiński [4], the Lebesgue measurability of a Jensen-convex function also implies its continuity. In this talk, we present a generalization of these theorems for (M, N)-convex functions, calling a function $f : I \to J$ (M, N)-convex (cf., e.g., [3]) if it satisfies the inequality $f(M(x, y)) \leq N(f(x), f(y))$ for all $x, y \in I$, where I and J are open intervals, M and N are suitable means on I and J, respectively. Our statements contain Tomasz Zgraja's results on (M, M)-convex functions (cf. [5]) as special cases. They generalize the main theorems presented in [2] as well.

References:

[1] F. Bernstein and G. Doetsch, *Zur Theorie der konvexen Funktionen*, Math. Ann. **76** (1915), 514–526.

[2] A. Gilányi and Zs. Páles, *Bernstein–Doetsch and Sierpiński theorems for (M, N)-convex functions*, Talk, 11^{th} International Conference on Functional Equations and Inequalities, Stefan Banach International Mathematical Center, Będlewo, Poland, September 17–23, 2006.

[3] C. Niculescu and L.-E. Persson, *Convex Functions and Their Applications*, CMS Books in Mathematics, Springer, 2006.

[4] W. Sierpiński, *Sur les fonctions convexes mesurables*, Fund. Math. **1** (1920), 125–128.

[5] T. Zgraja, *Continuity of functions which are convex with respect to means*, Publ. Math. **63** (2003), 401–411.

◇ ◇ ◇

Remarks on invariance equations involving generalized classical weighted means

Dorota Głazowska
University of Zielona Góra, Poland

Under some simple conditions on the real functions f, g and h defined on an interval $I \subset \mathbb{R}$ or $I \subset (0,\infty)$, the bivariable functions A_f, G_g and H_h given, respectively, by

$$A_f(x,y) = f(x) + y - f(y), \quad G_g(x,y) = \frac{g(x)}{g(y)}y, \quad H_h(x,y) = \frac{xy}{x - h(x) + h(y)},$$

are means in the interval I. These means are natural generalization, respectively, of the classical weighted arithmetic mean, the classical weighted geometric mean and the classical weighted harmonic mean.

During the talk, the solutions of some invariance problems involving these means will be presented.

This is a report on the research made jointly with Janusz Matkowski.

◇ ◇ ◇

Iterative roots of multidimensional maps

Chaitanya Gopalakrishna
Indian Statistical Institute, Bangalore, India

An *iterative root* of order $n \geq 2$ of a self-map f on a nonempty set is a self-map g on the set such that $g^n = f$. Most results on the iterative roots of continuous interval maps rely on tools such as the Intermediate Value Theorem and the monotonicity of bijections. Extending these results to higher dimensions or more general topological spaces is generally complex. In this talk we present some new combinatorial ideas that lead to new results on iterative roots of maps on arbitrary sets and continuous maps on topological spaces. These results, in particular, allow us to generalize some notable findings on iterative roots for continuous interval maps to the broader context of continuous multidimensional maps. The talk is based on our recent works [1, 2].

References:

[1] B. V. R. Bhat and C. Gopalakrishna, Iterative square roots of functions, *Ergodic Theory Dynam. Systems*, 43 (2023), 2201–2227.

[2] B. V. R. Bhat and C. Gopalakrishna, The non-iterates are dense in the space of continuous self-maps, *Nonlinearity*, 36 (2023), 3419–3430.

◇ ◇ ◇

On a general inequality in the theory of means

Richárd Grünwald
University of Debrecen, Hungary
(joint work with **Zsolt Páles**)

We are going to derive necessary conditions for the local validity of the functional inequality

$$M_0(\Phi(x_1), \dots, \Phi(x_n)) \leq \Phi(M_1(x^1), \dots, M_k(x^k)), \tag{1}$$

where $n, k \in \mathbb{N}$, for $\alpha \in \{0, \dots, k\}$, $I_\alpha \subseteq \mathbb{R}$ is a nonempty open interval, $I := I_1 \times \cdots \times I_k$, $M_\alpha \colon I_\alpha^n \to I_\alpha$ is an n-variable mean, $x^\alpha \in I_\alpha^n$ and $\Phi \colon I \to I_0$. If there exists an open set $U \subseteq I^n$ such that $\operatorname{diag}(I^n) \subseteq U$ and (1) holds for all $x \in U^T \subseteq \prod_{\alpha=1}^k I_\alpha^n$, then we say that (1) holds in the local sense. Here the *diagonal* $\operatorname{diag}(I^n)$ of I^n is defined by

$$\operatorname{diag}(I^n) := \{(x, \dots, x) \in \mathbb{R}^n \mid x \in I\}.$$

References:

[1] R. Grünwald and Zs. Páles, *Local and global Hölder- and Minkowski-type inequalities for nonsymmetric generalized Bajraktarević means*,
J. Math. Anal. Appl. **535** (2024), no. 128214.

◇ ◇ ◇

The compatibility of Weber's law and Iverson's law of similarity

Eszter Gselmann
University of Debrecen, Hungary
(joint work with **Christopher W. Doble and Yung-Fong Hsu**)

Weber's law states that the just noticeable difference (JND) between stimuli is proportional to the original stimulus's magnitude. While foundational in psychophysics, the law has limitations. It often fails at very low stimulus intensities, where changes may go undetected. For instance, doubling a faint sound may still be imperceptible. At very high intensities, sensory saturation can also disrupt proportional perception. Similarly, increasing an already bright light might not be noticed. These deviations challenge the law's universal applicability. Following Falmagne (1985), we derive Weber functions from sensitivity functions. We argue that many studies assume overly specific forms of these functions. This assumption may partly explain why Weber's law fails in some perceptual contexts.

◇ ◇ ◇

Refining the integral Jensen inequality for finite signed measures using majorization

László Horváth
Department of Mathematics, University of Pannonia, Hungary

Among inequalities that use the concept of convexity, Jensen-type inequalities and majorization-type inequalities are significant and fundamental. An important and widely researched area in the study of Jensen-type inequalities is the refinement of such inequalities. In this talk, we provide a general method for refining the integral Jensen inequality for finite signed measures using integral majorization inequalities. Under the conditions considered, the results are unique, and even for measures, they give a new approach. We also provide interesting specific refinements, some of which relate to Jensen–Steffensen's inequality.

◇ ◇ ◇

Quadratic functions as solution of polynomial equations

Mehak Iqbal
University of Debrecen, Hungary
(joint work with **Eszter Gselmann**)

In the theory of functional equations, many results are known when the unknown functions are assumed to be additive. In such cases, classical results show that the solutions are often homomorphisms, derivations, or combinations of these.

This raises a new question: what happens if we drop the assumption of additivity and instead assume that the unknown functions are generalized monomials? As a first step, we focus on monomials of degree two i.e., quadratic functions.

Let $\mathbb{K}$ be a field of characteristic zero and $\mathbb{F} \subset \mathbb{K}$ be a subfield of $\mathbb{K}$. Our main objective is to determine all those quadratic functions $q : \mathbb{F} \to \mathbb{K}$ that satisfy a Levi-Civita equation on the multiplicative structure.

To do this, we first examine quadratic functions q that satisfy the equations:

$$q(x, y) = q(x)q(y) \quad (x, y \in \mathbb{F}^{\times})$$

and

$$q(x, y) = x^2 q(y) + q(x)y^2 \quad (x, y \in \mathbb{F}^{\times}),$$

respectively.

◇ ◇ ◇

COMPLEX-VALUED SOLUTIONS OF A MULTIPLICATIVE EQUATION OF FINITE ORDER

Justyna Jarczyk
Institute of Mathematics, University of Zielona Góra, Poland
(joint work with **Witold Jarczyk**)

I will report a progress in studying solutions of the equation

$$\psi(x) = \prod_{j=1}^{n} \psi\left(f_j(x)\right)^{p_j(x)}$$

mapping a right vicinity $(0, a)$ of 0 into the complex plane and having a prescribed asymptotics at 0. We assume that $f_1, \ldots, f_n : (0, a) \to (0, a)$ are continuous functions satisfying the conditions

$$0 < f_j(x) < x, \quad x \in (0, a), \; j = 1, \ldots, n,$$

and $p_1, \ldots, p_n$ map (0,a) into positive half-line.

◇ ◇ ◇

ON ITERATIVE ROOTS OF MEAN-TYPE MAPPINGS

Witold Jarczyk
Institute of Mathematics, University of Zielona Góra, Poland

During the presentation some results concerning mean-type mappings and iteration together (but not dealing with the Gaussian algorithm!) will be recalled. I will remind also some questions still waiting for an answer and put new ones.

◇ ◇ ◇

ON ULAM STABILITY FOR OPERATOR RECURRENCE EQUATIONS

Delia-Maria Kerekes
Technical University of Cluj-Napoca, Romania
(joint work with **Dorian Popa**)

In this paper we investigate the Ulam stability for an operatorial recurrence of the form $x_{n+1} = T_n x_n + a_n$, where $T_n : X \to X$, $n \in \mathbb{N}$, are linear and bounded operators acting on a Banach space X. We analyze how perturbations affect the behavior of solutions, establishing conditions under which approximate solutions remain close to exact solutions. Our results yield explicit stability bounds for important classes of operators including Volterra, Fredholm and Gram-Schmidt operators, thereby extending and refining known results in this area.

REFERENCES:

[1] D. Popa, I. Rașa, *On the stability of some classical operators from approximation theory*, Expo. Math. 31 (2013), 205–214.

◇ ◇ ◇

Partially affine solutions of the invariance problem of Matkowski means

Tibor Kiss
University of Debrecen, Hungary
(joint work with **Péter Tóth**)

In the presentation, we focus on the composite functional equation

$$F\left(\frac{x+y}{2}\right) + f_1(x) + f_2(y) = G(g_1(x) + g_2(y)), \qquad (x, y \in I),$$

where $I \subseteq \mathbb{R}$ is a nonempty, open interval and $F, f_k, g_k : I \to \mathbb{R}$, $k = 1, 2$ and $G : g_1(I) + g_2(I) \to \mathbb{R}$ are considered unknown functions.

In the past, the above equation has been studied under various regularity conditions. First, we review these results. Then, we discuss a special family of solutions, known as partially affine solutions. To derive the results, we assume only the differentiability of the unknown functions, which, compared to previous results, is considered the weakest possible assumption.

◇ ◇ ◇

New upper bounds for the weighted Chebyshev functional

Milica Klaričić Bakula
University of Split, Faculty of Science, Croatia

We present some new upper bounds for the weighted Chebyshev functional under various conditions, including those of Steffensen type. We show how those results can be used to establish some new bounds for the Jensen functional.

◇ ◇ ◇

On the form of the set of irregular points for a class of Brouwer homeomorphisms

Zbigniew Leśniak
Department of Mathematics, University of the National Education Commission, Krakow, Poland

We present properties of the set of irregular points of a Brouwer homeomorphism f for which there exists a foliation of the plane whose leaves are invariant lines of f. For such a homeomorphism, we show that the limit set of any irregular point p can be obtained as the limit of iterates of a Jordan domain containing p in its interior.

References:

[1] F. Le Roux, A.G. O'Farrell, M. Roginskaya and I. Short, *Flowability of plane homeomorphisms*, Ann. Inst. Fourier Grenoble **62** (2012), 619–639.

[2] Z. Leśniak, *On families of invariant lines of a Brouwer homeomorphism*, J. Difference Equ. Appl. **25** (2019), 1363–1371.

◇ ◇ ◇

Probability distributions and their entropies

Alexandra Măduța

Technical University of Cluj-Napoca, Department of Mathematics, Romania

(joint work with **Ioan Rașa**)

This talk explores the connection between probability distributions and their associated entropies, both in discrete and continuous settings. Classical and new examples of positive linear operators are discussed, along with entropy measures such as Rényi, Tsallis, and Shannon. The presentation also highlights two open problems related to convexity and positivity, proposed by Ioan Rașa.

⋄ ⋄ ⋄

Hyers-Ulam stability of certain partial differential equations

Daniela Marian

Technical University of Cluj-Napoca, Department of Mathematics, Romania

We present some results regarding Hyers-Ulam and Hyers-Ulam-Rassias stability of certain partial differential equations. We also give some examples.

References:

[1] D. Marian, S. A. Ciplea, N. Lungu, *Hyers-Ulam stability of a nonlinear partial integro-differential equation of order three,* Open Mathematics, **22** (1), 2024, pp. 20240017.

[2] D. Marian, S. A. Ciplea, N. Lungu and Th. M. Rassias,*Hyers-Ulam stability of a general linear partial differential equation,* Aequat. Math. 97, (2023), 649–657. https://doi.org/10.1007/s00010-023-00960-3

[3] N. Lungu and D. Marian, *Hyers-Ulam-Rassias stability of some quasilinear partial differential equations of first order,* Carpathian J. Math. **35** (2019) , no. 2, 165–170.

[4] D. Marian, S. A. Ciplea and N. Lungu, *Hyers-Ulam stability of Darboux-Ionescu problem,* Carpatian J. Math. **37** (2021), no. 2, 211–216.

[5] A. Prastaro and Th. M. Rassias, *Ulam stability in geometry of PDE's,* Nonlinear Funct. Anal. Appl. **8** (2003), no. 2, 259–278.

⋄ ⋄ ⋄

Rates of convergence for positive linear operators

Gabriela Motronea
Technical University of Cluj-Napoca, Romania

We are concerned with positive linear operators defined on $C(X)$, where X is a simplex or a hypercube. We assume that the operators preserve the affine functions. After identifying an eigenvalue $a \in [0,1)$ of such an operator L, we show that the sequence $(L^k f)_{k\geq 1}$ has a limit Vf, $f \in C(X)$ and $|L^k f(x) - Vf(x)|$ is dominated by a^k multiplied by a factor depending on L, f and x.

References:

[1] AM Acu, G. Bascanbaz-Tunca, I Rasa, *Differences of positive linear operators on simplices*, Journal of Function Spaces, Article Number 5531577, 2021.

[2] Acu, AM., Heilmann, M., Rasa, I. Eigenstructure and iterates for uniquely ergodic Kantorovich modifications of operators II. Positivity 25, 1585–1599 (2021).

[3] Acu, AM., Heilmann, M., Rasa, I. Iterates of convolution-type operators. Positivity 25, 495–506 (2021).

[4] O. Agratini, R. Precup, Iterates of multidimensional approximation operators via Perov theorem. (English) Zbl 07752838 Carpathian J. Math. 38, No. 3, 539-546 (2022).

[5] O. Agratini, I. Rus, Iterates of a class of discrete linear operators via contraction principle. Commentat. Math. Univ. Carol. 44, No. 3, 555-563 (2003).

[6] Altomare, M. Campiti, Korovkin-type Approximation Theory and its Applications, Series: De Gruyter Studies in Mathematics, 17, 1994.

◇ ◇ ◇

Old and new on strongly subadditive/superadditive functions

Constantin P. Niculescu
University of Craiova, Romania

A real-valued function Φ defined on a convex cone $\mathcal{C}$ of a real linear space E is called *strongly subadditive* if $\Phi(0) \geq 0$ and

$$\Phi(x+y+z) + \Phi(z) \leq \Phi(x+z) + \Phi(y+z),$$

for all $x, y, z \in \mathcal{C}$. The condition $\Phi(0) \geq 0$ was added to assure that a strong subadditive function is also subadditive.

The aim of my talk is to reveal the existence of a wide range of strongly subadditive/superadditive functions, motivated by potential theory, probability theory and statistics, combinatorial optimization, risk management, quantum physics, etc. As a basis we used the updated 2025 version of the book [1], published in collaboration with Lars-Erik Persson.

References:

[1] Niculescu, C.P., Persson, L.-E.: Convex Functions and Their Applications. A Contemporary Approach, 3rd Ed. CMS Books in Mathematics Vol. 23, Springer-Verlag, New York (2025)

◇ ◇ ◇

On expectations of power means of random variables

Kazuki Okamura

Shizuoka University, Japan

(joint work with **Yoshiki Otobe**)

In general, it is hard to compute or characterize the expectation of a power mean of independent and identically distributed random variables, due to the fractional power of the mean. We investigate expectations of power means of complex-valued random variables by using fractional calculus. We explicitly compute the expectations of the power means for both the univariate Cauchy distribution and the Poincaré distribution on the upper-half plane. We show that for these distributions the expectations are invariant with respect to the sample size and the value of the power. This talk will depend on [1].

References:

[1] K. Okamura and Y. Otobe, Power means of random variables and characterizations of distributions via fractional calculus, Probability and Mathematical Statistics (2024) vol.44, Fasc.1, 133-156.

◇ ◇ ◇

Best Ulam constant of a partial differential operator

Diana Otrocol

Technical University of Cluj-Napoca, Department of Mathematics, Romania

(joint work with **Adela Novac, Dorian Popa, Alexandra Sîngeorzan**)

In this paper we give a characterization of Ulam stability for the linear partial differential operator $D\colon C^1(\mathbb{R}^2, X) \to C(\mathbb{R}^2, X)$ given by $Du = au_x + bu_y + cu$, where $a, b, c \in \mathbb{R}$ and X is a Banach space over $\mathbb{R}$. Moreover we obtain the best Ulam constant of the operator D.

References:

[1] Brzdek, J.; Popa, D.; Rașa, I.; Xu, B., *Ulam Stability of Operators*, Academic Press: Cambridge, MA, USA, 2018.

[2] Lungu, N.; Popa, D., *Hyers-Ulam stability of a first order partial differential equation*, J. Math. Anal. Appl., 2012, 385, 86–91.

[3] Novac, A.; Otrocol, D.; Popa, D., *On Ulam stability of a partial differential operator in Banach spaces*, Mathematics, 2023, 11, 2488.

◇ ◇ ◇

Results and problems in the comparison theory of means

Zsolt Páles
Institute of Mathematics, University of Debrecen, Hungary

In the theory of means several classes were introduced and discovered in the last 100 years. Power means and then quasiarithmetic means were the first classes which were studied systematically in the thirties of the last century. In the subsequent 50 years Gini, Bajraktarević, Daróczy, Stolarsky, Leech and Sholander and Losonczi introduced further new classes of means. Some of these means are defined in terms of real (or complex) parameters, other means are constructed via one or two variable functions. The process of generalizations of means has not terminated yet. Matkowski, Pasteczka and the author of this talk created further interesting classes in the last 20 years. In the talk we are going to recall the most interesting results and describe the most challenging open problems.

◇ ◇ ◇

Perspectives for the mean theory: open problems and further developments

Paweł Pasteczka
University of the National Education Commission, Krakow, Poland

The aim of this talk is to present potential developments of the mean theory. We show open problems, which were also posed by other authors.

◇ ◇ ◇

Characterizations of the limits of certain sequences of positive linear operators

Ioan Rașa
Technical University of Cluj-Napoca, Romania
(joint work with **Ana-Maria Acu and Gabriela Motronea**)

Inspired by the Poisson approximation to the binomial distribution, several sequences of positive linear operators were constructed, having as limits operators different from the identity (see [1] and references therein). The limit operators can be characterized in terms of suitable functional equations. We present several results in this direction.

References:

[1] A.M. Acu, M. Heilmann, I. Rasa, A. Seserman, Poisson approximation to the binomial distribution: extensions to the convergence of positive operators. Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat. 117, 162 (2023).

◇ ◇ ◇

On supertranslativity of the Orlicz premium principle under uncertainty

Patryk Rela
University of Rzeszów, Poland

The Orlicz premium principle under uncertainty for the risk X, represented by non-negative, essentially bounded from above measurable function on a given measurable space $(\Omega, \mathcal{F})$, is defined as a solution $H_{(\mu,\alpha,\Phi)}(X)$ of the equation

$$E_\mu \left[\Phi \left(\frac{X}{H_{(\mu,\alpha,\Phi)}(X)} \right) \right] = 1 - \alpha,$$

where $\Phi : [0, \infty) \to [0, \infty)$ is a normalized Young function, $\alpha \in [0, 1)$ is a given parameter and E_μ is a Choquet integral with respect to capacity μ. The aim of this talk is to establish a characerization of normalized Young functions that generate supertranslative Orlicz premium principles.

References:

[1] Haezendonck J., Goovaerts M., A new premium calculation principle based on Orlicz norms, Insur Math Econ 1 (1982), 41-53.

[2] Chudziak, J., Rela, P.: On supertranslativity of the Orlicz premium principle.

◇ ◇ ◇

Consistence of social indices

Maciej Sablik
University of Silesia, Poland

We are discussing the UNDP social indices and their consistency, in particular Human Poverty Index. We are using results on generalized bisymmetry obtained by J. Aczél, Gy. Maksa, M. Taylor, A. Muennich and R. J. Mokken.

References:

[1] J. Aczél, G. Maksa, M. A. Taylor, *Equations of generalized bisymmetry and of consistent aggregation: Weakly surjective solutions which may be discontinuous at places.* J. Math. Anal. Appl. *214*(1997), 22-35.

[2] G. Maksa, *Solution of generalized bisymmetry equations without surjectivity assumptions.* Aequationes Math. *57*(1999), 50-74.

[3] A. Münnich, G. Maksa, R. J. Mokken, *n-variable bisection*, J. Math. Psych. 44 (2000), no. 4, 569-581.

◇ ◇ ◇

Localization of ideals in the Fourier algebra

László Székelyhidi
University of Debrecen, Hungary

Recently we introduced the concept of localization of ideals in the Fourier algebra of a locally compact abelian group. Roughly speaking, an ideal is localizable, if it is completely determined by the set of differential operators, which annihilate it at the zeros of the ideal. It turns out that localizability is equivalent to synthesizability. Using this fact we are able to characterize those locally compact abelian groups which have spectral synthesis.

⋄ ⋄ ⋄

Further results concerning the computer-assisted approach to inequalities

Tomasz Szostok
University of Silesia, Poland
(joint work with **Chisom P. Okeke**)

We continue the research from the paper [1]. Namely, we construct a computer program solving the inequality between two quadratures (in the case where these quadratures are comparable in the class of higher-order convex functions of some order). We also present some remarks concerning the possible application of computer programs to obtaining results of Ostrowski type.

References:

[1] T. Nadhomi, C.P. Okeke, M. Sablik, T. Szostok *On a class of functional inequalities, a computer approach* Period. Math. Hungar. (2025). https://doi.org/10.1007/s10998-025-00632-6

⋄ ⋄ ⋄

Computer-assisted investigation of types of functional equations

Lan Nhi To
University of Nyiregyhaza, Hungary
(joint work with **Attila Gilányi**)

Extending the results in [1] and [2], we present a computer programs package, developed in the computer algebra system MAPLE, which determines types or classes of functional equations. Additionally, we introduce a GUI-based function that shows theoretical information for certain types and classes of functional equations.

References:

[1] L. N. To. 2.9. Remark, Report of Meeting, Aequationes Math. 97 (2023), 1287.

[2] A. Gilányi. On a computer program, which determines types of functional equations, Report of Meeting, Aequationes Math. 98 (2024), 1693.

⋄ ⋄ ⋄

Testing non-solvability of linear programs

Norbert Tóth

Institute of Mathematics, Faculty of Science and Technology, University of Debrecen, Hungary

(joint work with **Mihály Bessenyei**)

It may occur that only the solvability issue of a linear program counts in a practical application (and the solution itself, if it exists, does not). Motivated by this phenomena, we suggest an implementable method for testing non-solvability of linear programs.

◇ ◇ ◇

Smooth solutions of the invariance problem of Matkowski means

Péter Tóth

University of Debrecen, Hungary

(joint work with **Tibor Kiss**)

We investigate the functional equation

$$F\left(\frac{x+y}{2}\right)+f_1(x)+f_2(y)=G(g_1(x)+g_2(y)) \qquad (x,y\in I)$$

supposing that $\emptyset\neq I\subseteq\mathbb{R}$ is an open interval and the unknown functions $F, f_1, f_2, g_1, g_2 : I \longrightarrow \mathbb{R}$ and $G : g_1(I)+g_2(I)\longrightarrow\mathbb{R}$ are differentiable. We further assume that g_1, g_2 are strictly monotone in the same sense and that F is not affine on any open subinterval. Introducing the notations

$$\varphi:=\frac{1}{2}F', \qquad \psi_k:=\frac{1}{g_1'}+(-1)^k\frac{1}{g_2'}, \qquad \Psi_k:=-\frac{f_1'}{g_1'}-(-1)^k\frac{f_2'}{g_2'} \qquad (k=1,2)$$

we solve the system of functional equations

$$\begin{cases}\varphi\left(\frac{x+y}{2}\right)(\psi_1(x)+\psi_1(y))=\Psi_1(x)+\Psi_1(y)\\ \varphi\left(\frac{x+y}{2}\right)(\psi_2(x)-\psi_2(y))=\Psi_2(x)-\Psi_2(y)\end{cases} \qquad (x,y\in I).$$

◇ ◇ ◇

On some functional equations of function of natural argument and the convergence of the solutions

Andrei Vernescu

University of Târgoviște, Romania

Starting from some examples of Alexandru Lupaș, we continue to show for certain functional equations in which the sought function is of natural argument, i.e.for some recurrence relations, not the influence of the initial condition about the convergence, but conversely, how the recurrence relation, together with a convergence condition, determine the initial condition. We also characterize the speed of convergence of some of these sequences.

◇ ◇ ◇

Personal Notes