62nd International Symposium on Functional Equations

June 14–20 2026, Hajdúszoboszló, Hungary

Dedicated to the memory of Professor Roman Ger

Organized by the
Institute of Mathematics, University of Debrecen

UNIVERSITY of DEBRECEN

Book of Abstracts

Scientific Committee

- Jacek Chmieliński (University of the National Education Commission, Poland)
- Gian Luigi Forti (University of Milan, Italy)
- Eszter Gselmann (University of Debrecen, Hungary)
- Zsolt Páles (University of Debrecen, Hungary)
- Maciej Sablik (University of Silesia, Poland) – chairman
- Jens Schwaiger (University of Graz, Austria)
- Ekaterina Shulman (University of Silesia, Poland)
- László Székelyhidi (University of Debrecen, Hungary)
- Ludwig Reich (University of Graz, Austria) – honorary member.

Organizing Committee

- Zsolt Páles (University of Debrecen, Hungary) – chairman
- Attila Gilányi (University of Debrecen, Hungary) – co-chairman
- Zoltán Boros (University of Debrecen, Hungary) – co-chairman
- Eszter Gselmann (University of Debrecen, Hungary)
- Richárd Grünwald (University of Debrecen, Hungary) – technical secretary
- Péter Tóth (University of Debrecen, Hungary) – scientific secretary
- Paweł Solarz (University of the National Education Commission, Poland) – web administrator.

General Information

The conference is held in Hotel Aurum in Hajdúszoboszló, Hungary, from Sunday, June 14 (arrival day) to Saturday, June 20, 2026 (departure day). The participation fee covers full board, accommodation, the use of the spa and the sauna in Hotel Aurum, registration materials, the fees of the excursion, the banquet and other social events of the meeting.

All conference *talks* are given in the *Main Lecture Room*, which is equipped with a computer, a data projector, and a blackboard. The Main Lecture Room and the Coffee Breaks are in the Park Hotel Ambrózia. The duration of every regular talk is at most 20 minutes, which is followed by a discussion of at most 5 minutes. There are no breaks between the talks within a session, therefore the schedule of the individual talks is only approximative. Speakers cannot inherit time from the previous talk. The time saved by shorter talks can be devoted to problems and remarks at the end of the session. If you have special wishes concerning the schedule, you are welcome to consult Péter Tóth, the scientific secretary of the symposium.

During the conference, you have the possibility to photocopy and print a limited number of pages in the Conference Office. Internet is also available in the hotel with a wireless connection.

You can find the program and the abstracts in this booklet. The schedule of the individual sessions and further technical details are announced during the conference. Your questions may help the Organizing Committee to improve organization, so do not hesitate to contact us. We hope that our conference will be interesting and successful and you will enjoy your stay in Hajdúszoboszló.

62nd International Symposium on Functional Equations

Hajdúszoboszló, Hungary, June 14–20, 2026

Program

		Sunday	
		18:00–20:30	Dinner

Monday		**Tuesday**	
07:30–09:00	Breakfast	07:30–09:00	Breakfast
09:00–09:15	Opening		
09:15–10:30	1st Morning Session	09:00–09:55	1st Morning Session
10:30–11:20	*Coffee Break*	09:55–10:45	*Coffee Break*
11:20–12:10	2nd Morning Session	10:45–11:35	2nd Morning Session
12:15–14:00	Lunch	12:15–14:00	Lunch
15:00–15:50	1st Afternoon Session	15:00–15:50	1st Afternoon Session
15:50–16:45	*Coffee Break*	15:50–16:45	*Coffee Break*
16:45–18:00	2nd Afternoon Session	16:45–18:00	2nd Afternoon Session
18:00–20:30	Dinner	18:00–20:30	Dinner

Wednesday		**Thursday**	
07:30–09:00	Breakfast	07:30–09:00	Breakfast
		Special Session	
09:00–09:50	1st Morning Session	09:00–10:15	1st Morning Session
09:50–10:45	*Coffee Break*	10:15–11:20	*Coffee Break*
10:45–12:00	2nd Morning Session	11:20–12:35	2nd Morning Session
12:00–13:00	Lunch	12:40–14:00	Lunch
13:00–	Excursion to Hortobágy, returning back to the Hotel after dinner	15:00–16:15	1st Afternoon Session
		16:15–16:55	*Coffee Break*
		16:55–18:30	2nd Afternoon Session
		19:30–23:00	Banquet

Friday		**Saturday**	
07:30–09:00	Breakfast	07:30–09:00	Breakfast
09:00–09:50	1st Morning Session		
09:50–10:45	*Coffee Break*		
10:45–11:35	2nd Morning Session		
12:15–14:00	Lunch		
15:00–16:30	1st Afternoon Session		
16:30–17:30	*Coffee Break*		
17:30–18:30	Medal Ceremony, Closing		
18:30–20:30	Dinner		

List of Participants

1. Mátyás BARCZY, University of Szeged, Szeged, Hungary,
   email: *barczy@math.u-szeged.hu*
2. Mihály BESSENYEI, University of Miskolc, Miskolc, Hungary,
   email: *mihaly.bessenyei@uni-miskolc.hu*
3. Zoltán BOROS, University of Debrecen, Debrecen, Hungary,
   email: *zboros@science.unideb.hu*
4. Jacek CHMIELIŃSKI, University of the National Education Commission, Kraków, Poland,
   email: *jacek.chmielinski@uken.krakow.pl*
5. Jacek CHUDZIAK, University of Rzeszów, Rzeszów, Poland,
   email: *jchudziak@ur.edu.pl*
6. Gregory DERFEL, Ben Gurion University, Beer-Sheva, Israel,
   email: *derfel@math.bgu.ac.il*
7. Wolfgang FÖRG-ROB, University of Innsbruck, Innsbruck, Austria,
   email: *wolfgang.foerg-rob@uibk.ac.at*
8. Gian Luigi FORTI, University of Milan, Milano, Italy,
   email: *gianluigi.forti@unimi.it*
9. Harald FRIPERTINGER, University of Graz, Graz, Austria,
   email: *harald.fripertinger@uni-graz.at*
10. Attila GILÁNYI, University of Debrecen, Debrecen, Hungary,
    email: *gilanyi@inf.unideb.hu*
11. Dorota GŁAZOWSKA, University of Zielona Góra, Zielona Góra, Poland,
    email: *d.glazowska@im.uz.zgora.pl*
12. Chaitanya GOPALAKRISHNA, Indian Statistical Institute, Bengaluru, India,
    email: *cberbalaje@gmail.com*
13. Angshuman Robin GOSWAMI, University of Pannonia, Veszprém, Hungary,
    email: *goswami.angshuman.robin@mik.uni-pannon.hu*
14. Richárd GRÜNWALD, University of Debrecen, Debrecen, Hungary,
    email: *richard.grunwald@science.unideb.hu*
15. Eszter GSELMANN, University of Debrecen, Debrecen, Hungary,
    email: *gselmann@science.unideb.hu*
16. László HORVÁTH, University of Pannonia, Veszprém, Hungary,
    email: *horvath.laszlo@mik.uni-pannon.hu*

17. Szymon IGNACIUK, University of Life Sciences in Lublin, Lublin, Poland,
email: *szymon.ignaciuk@up.lublin.pl*

18. Daniela INOAN, Technical University of Cluj-Napoca, Cluj-Napoca, Romania,
email: *daniela.inoan@math.utcluj.ro*

19. Justyna JARCZYK, University of Zielona Góra, Zielona Góra, Poland,
email: *j.jarczyk@im.uz.zgora.pl*

20. Witold JARCZYK, University of Zielona Góra, Zielona Góra, Poland,
email: *w.jarczyk@im.uz.zgora.pl*

21. Tibor KISS, University of Debrecen, Debrecen, Hungary,
email: *kiss.tibor@science.unideb.hu*

22. Gergely KISS, Alfréd Rényi Institute of Mathematics and Corvinus University of Budapest, Budapest, Hungary,
email: *kigergo57@gmail.com*

23. Paweł KLUZA, University of Life Sciences in Lublin, Lublin, Poland,
email: *pawel.kluza@up.lublin.pl*

24. Zbigniew LEŚNIAK, University of the National Education Commission, Kraków, Poland,
email: *zbigniew.lesniak@uken.krakow.pl*

25. Judit MAKÓ, University of Miskolc, Miskolc, Hungary,
email: *judit.mako@uni-miskolc.hu*

26. Gábor MOLNÁR, University of Nyíregyháza; University of Debrecen, Nyíregyháza; Debrecen, Hungary,
email: *molnar.gabor@nye.hu*

27. Constantin NICULESCU, University of Craiova, Craiova, Romania,
email: *constantin.p.niculescu@gmail.com*

28. Kazuki OKAMURA, Shizuoka University, Shizuoka, Japan,
email: *okamura.kazuki@shizuoka.ac.jp*

29. Zsolt PÁLES, University of Debrecen, Debrecen, Hungary,
email: *pales@science.unideb.hu*

30. Paweł PASTECZKA, University of Rzeszów, Rzeszów, Poland,
email: *ppasteczka@ur.edu.pl*

31. Ludwig REICH, University of Graz, Graz, Austria,
email: *ludwig.reich@uni-graz.at*

32. Patryk RELA, University of Rzeszów, Rzeszów, Poland,
email: *prela@ur.edu.pl*

33. Maciej SABLIK, University of Silesia, Katowice, Poland,
email: *maciej.sablik@us.edu.pl*

34. Paweł SOLARZ, University of the National Education Commission, Kraków, Poland,
email: *pawel.solarz@up.krakow.pl*

35. Rafał STYPKA, University of the National Education Commission, Kraków, Poland,
email: *rafal.stypka@uken.krakow.pl*

36. László SZÉKELYHIDI, University of Debrecen, Debrecen, Hungary,
email: *lszekelyhidi@gmail.com*

37. Tomasz SZOSTOK, University of Silesia, Katowice, Poland,
email: *tomasz.szostok@us.edu.pl*

38. Péter TÓTH, University of Debrecen, Debrecen, Hungary,
email: *toth.peter@science.unideb.hu*

39. Andrei VERNESCU, Valahia University, Târgoviște, Romania,
email: *avernescu@gmail.com*

List of Talks

1. Mátyás BARCZY, *Axiomatic characterisation of generalized ψ-estimators*,
2. Mihály BESSENYEI, *Equivalent contraction principles in metric spaces*,
3. Zoltán BOROS, *Separable non-negativity on big sets*,
4. Jacek CHMIELIŃSKI, *Approximate rotundity in normed linear spaces*,
5. Jacek CHUDZIAK, *On properties of the optimized certainty equivalent*,
6. Gregory DERFEL, *On the balanced pantograph equation of mixed type*,
7. Wolfgang FÖRG-ROB, *Means and convergent sequences*,
8. Gian Luigi FORTI, *Mikusiński's alternative equation on semigroups*,
9. Harald FRIPERTINGER, *Maximal Families of Commuting Power Series of Type II*,
10. Attila GILÁNYI, *On conditional linear functional equations*,
11. Dorota GŁAZOWSKA, *Weakly associative Bajractarević means*,
12. Chaitanya GOPALAKRISHNA, *On iterative roots of some multifunctions*,
13. Angshuman Robin GOSWAMI, *On Weighted Convex Graphs*,
14. Eszter GSELMANN, *When are solution spaces finite-dimensional? Linear differential-algebraic equations revisited*,
15. László HORVÁTH, *Exact conditions for discrete refinements on both sides of the Hermite-Hadamard inequality*,
16. Szymon IGNACIUK, *Distributed Halpern Iteration: Strong Convergence to a Common Fixed Point*,
17. Daniela INOAN, *On selections of generalized convex set-valued maps*,
18. Justyna JARCZYK, *Iterates of Möbius transformations and their dynamics*,
19. Witold JARCZYK, *Recent results in the theory of means – a personal selection*,
20. Gergely KISS, *N-ary quasi-arithmetic means without regularity*,
21. Tibor KISS, *Strictly monotonic, strictly nonexpansive quasi graph-additive functions*,
22. Paweł KLUZA, *Bernstein–Doetsch Theorem for geometric convex functions*,
23. Zbigniew LEŚNIAK, *Iterative roots of Brouwer homeomorphisms and the four point matching property*,
24. Judit MAKÓ, *Approximate convexity and Bernstein–Doetsch type theorems*,
25. Gábor MOLNÁR, *On trigonometric φ-Jensen convex functions*,

26. Constantin NICULESCU, *On a class of nonlinear operators*,
27. Kazuki OKAMURA, *On graph-directed conjugacy equations indexed by a two-vertex digraph*,
28. Zsolt PÁLES, *Invariance equation for weighted two-variable Bajraktarević means*,
29. Paweł PASTECZKA, *When are local properties of solutions of iterative functional equations actually global?*,
30. Ludwig REICH, *Semicanonical Normal Forms and some Applications*,
31. Patryk RELA, *The robust Orlicz premium principle under uncertainty*,
32. Maciej SABLIK, *Roman – The Boss, The Master, The Friend*,
33. Rafał STYPKA, *Nonlinear equivalence of finite-dimensional normed spaces based on approximate Birkhoff–James orthogonality*,
34. László SZÉKELYHIDI, *Equivalence of Spectral Analysis and Spectral Synthesis*,
35. Tomasz SZOSTOK, *The difference property and higher-order convexity properties of difference operators*,
36. Péter TÓTH, *Generalized inverses of strictly monotone transformations*,
37. Andrei VERNESCU, *Some considerations on the equations $g \circ g = f$ in $C(\mathbb{R})$*,

Abstracts

Axiomatic characterisation of generalized ψ-estimators

Mátyás Barczy
University of Szeged, Szeged, Hungary

(joint work with Zsolt Páles, University of Debrecen, Debrecen, Hungary)

A generalized ψ-estimator is defined as a unique point of sign change of some appropriate function. We give an axiomatic characterisation of such estimators. The key properties of the estimators that come into play in the characterisation theorem are the symmetry, the (strong) internality and the asymptotic idempotency. In the proofs, a separation theorem for Abelian subsemigroups plays a crucial role. The talk is based on the paper [1].

References

[1] M. Barczy, Zs. Páles, *Axiomatic characterisation of generalized ψ-estimators*, Metrika (2026+). DOI: https://doi.org/10.1007/s00184-026-01023-8

Equivalent contraction principles in metric spaces

Mihály Bessenyei
University of Miskolc, Hungary

A branch of the generalization of the Banach fixed point theorem demonstrates that the unique existence of the fixed point still remains valid if we relax the notion of contractions. Although the literature on these types of results has now become almost incomprehensible, it has been turned out in many cases that certain seemingly different contraction principles are actually equivalent. We prove that this occurs exactly in case of the results of Boyd, Wong, Matkowski, Bessenyei, Browder, Hegedűs, Szilágyi and Walter. The key tool of our approach is to redefine the underlying metric properly.

Separable non-negativity on big sets

Zoltán Boros
University of Debrecen, Hungary
(joint work with Rayene Menzer)

Based on the arguments in [1], we investigate non-negativity of separable expressions on big subsets of product spaces in the following sense. Let k, $m \in \mathbb{N}$. Suppose that $f : \mathbb{R}^k \to \mathbb{R}$ and $g : \mathbb{R}^m \to \mathbb{R}$ satisfy $f(x)g(y) \geq 0$ for all $(x, y) \in D$, where $D \subseteq \mathbb{R}^{k+m}$ has a positive $k + m$ dimensional Lebesgue measure or D is a second category Baire set. Then either one of these functions vanishes on a big set or both functions keep their signs on big sets (in the corresponding sense).

References

[1] Z. Boros, R. Menzer, *An alternative equation for generalized polynomials involving measure and category constraints*, Acta Math. Hungar. **175** (2025), 376–394.

Approximate rotundity in normed linear spaces

Jacek Chmieliński
University of the National Education Commission, Krakow, Poland
(joint work with Sebastian Krupa)

Rotundity (strict convexity) is a fundamental geometric property of normed linear spaces. In the talk, we introduce its approximate version and provide several equivalent characterizations. We also present examples of approximately rotund spaces and investigate their properties.

On properties of the optimized certainty equivalent

Jacek Chudziak
Institute of Mathematics, University of Rzeszów, Poland
(joint work with Sabina Lizis and Patryk Rela)

Assume that $\mathcal{X}$ is a family of all bounded random variables on a given probability space. Let $\mathcal{U}$ be a family of all non-decreasing concave utility functions such that $u(0) = 0$ and $u(x) \leq x$ for $x \in \mathbb{R}$. For any $u \in \mathcal{U}$, the optimized certainty equivalent of $X \in \mathcal{X}$ is defined by (cf. [1])

$$S_u(X) = \sup\{t + E[u(X - t)] : t \in \mathbb{R}\}.$$

We deal with the problem of recovering of utility function from its optimized certainty equivalent. Moreover, we present some applications of our results.

References

[1] A. Ben-Tal, M. Teboulle, *Expected utility, penalty functions, and duality in stochastic nonlinear programming*, Management Sci. **32** (1986), 1445–1466.

On the balanced pantograph equation of mixed type

Gregory Derfel
Ben Gurion University, Israel

(joint work with Bruce van Brunt, Massey University, New Zeland)

We consider the balanced pantograph equation (BPE)

$$y'(x) + y(x) = \sum_{k=1}^{m} p_k y(a_k x),$$

where $a_k, p_k > 0$, and $\sum_{k=1}^{m} p_k = 1$. It is known that if $K = \sum_{k=1}^{m} p_k \ln a_k \leq 0$ then, under mild technical conditions, the BPE does not have bounded solutions that are not constant; whereas, if $K > 0$ (*the supercritical case*) such solutions exist. In the present paper we refine the above result for a BPE of the *mixed type* i.e. $a_1 < 1 < a_m$ in *the supercritical case*. Namely, we prove that in this case the BPE has a non-constant solution y such that $y(x) \sim cx^{-\sigma}$ as $x \to \infty$, where $c > 0$ and σ is the unique positive root of the characteristic equation $P(s) = 1 - \sum_{k=1}^{m} p_k a_k^{-s} = 0$. Moreover, y is unique (up to a multiplicative constant) among solutions of the BPE that decay to zero as $x \to \infty$.

References

[1] G.Derfel, B.van Brunt, *On the balanced pantograph equation of mixed type*, *Ukrainian Math. J.*, **75**/12 (2023), 1627–1634.

Means and convergent sequences

Wolfgang Förg-Rob
Institute of Mathematics, University of Innsbruck, Austria

We generalize a theorem of Housen Li and Markus Haltmeier (which was stated for the arithmetic mean) to the following result: Let M be a strict, continuous and monotonous k-mean on an interval. If each element of a sequence in this interval is bounded from above by the mean of the preceding k elements then the sequence is convergent.
Furthermore, we give counterexamples if the mean fails to fulfil the imposed conditions.

References

[1] Housen Li and Markus Haltmeier, *The Avaraged Kaczmarz Iteration for Solving Inverse Problems*, SIAM Journal on Imaging Sciences **11**/1 (2018), doi.org/10.1137/17M1146178

Mikusiński's alternative equation on semigroups

Dedicated to the memory of Roman Ger

Gian Luigi Forti
Università degli Studi di Milano, Italy

Let $(G,\cdot)$ and $(H,\cdot)$ be two groups with identities e and ϵ respectively. The following alternative equation

$$f(xy) \neq \epsilon \text{ implies } f(xy) = f(x)f(y) \tag{E}$$

is known as Mikusiński equation. It was studied and completely solved in 1973 by L. Dubikajtis, C. Ferenc, R. Ger and M. Kuczma. They described the nontrivial (non multiplicative) solutions of equation (E).

Equation (E) needs not to be considered in groups, it is sufficient that G and H be semigroups with identity. We investigate this case producing examples of nontrivial solutions, when they exist, under various conditions on the semigroups and on the set where the function f assume the value ϵ.

References

[1] L. Dubikajtis, C. Ferenc, R. Ger, M. Kuczma: *On Mikusiński's functional equation*, Ann. Polon. Math. **28** (1973), 39-47.

Maximal Families of Commuting Power Series of Type II

Harald Fripertinger
Department of Mathematics and Scientific Computing, University of Graz, Austria
(joint work with Ludwig Reich)

Families of commuting invertible formal power series in one indeterminate over $\mathbb{C}$ are studied using the method of *formal functional* equations. We give a characterization of such families where the set of multipliers (first coefficients) σ of its members $F(x) = \sigma x + \dots$ is finite, in particular of such families which are maximal with respect to inclusion, so called families of *type II*. The description of these families is based on Aczél–Jabotinsky differential equations, iteration groups, and on some results on semicanonical forms of invertible series with respect to conjugation. Finally we study all maximal families of commuting power series into which a series $\sigma x + \dots$ can be embedded where σ is a complex root of unity.

On conditional linear functional equations

Dedicated to the memory of Roman Ger

ATTILA GILÁNYI
University of Debrecen, Hungary

Problems related to extensions and conditionally fulfilled properties often occur in the field of functional equations and their applications (cf., e.g., the 'Program' [1] announced by János Aczél, as well as [2] and [3]). Connected to this topic, some results on conditional linear functional equations are presented.

References

[1] J. Aczél, *5. Remark*, Report of meeting, Aequationes Math. **69** (2005), 183.
[2] R. Ger, *On almost polynomial functions*, Colloq. Math. **24** (1971), 95–101.
[3] R. Ger, *On some properties of polynomial functions*, Ann. Polon. Math. **25** (1971), 195–203.

Weakly associative Bajraktarević means

Dorota Głazowska
University of Zielona Góra, Poland

In the talk I am going to present some results concerning the problem of weak associativity of Bajraktarević means, i.e. the mean $B_{f,(p_1,p_2)}\colon I^2 \to I$ defined by

$$B_{f,(p_1,p_2)}(x,y) = f^{-1}\left(\frac{p_1(x)f(x)+p_2(y)f(y)}{p_1(x)+p_2(y)}\right),$$

where I is an open real interval, $f\colon I \to \mathbb{R}$ is a continuous and strictly monotone function and $p_1, p_2\colon I \to (0,+\infty)$ are weight functions. More precisely, I will report a progress in studying solutions of the functional equation

$$B_{f,(p_1,p_2)}\Big(B_{f,(p_1,p_2)}(x,y),x\Big) = B_{f,(p_1,p_2)}\Big(x, B_{f,(p_1,p_2)}(y,x)\Big), \qquad x,y \in I,$$

under certain additional regularity assumptions imposed on the unknown functions f, p_1, and p_2.

On iterative roots of some multifunctions

Chaitanya Gopalakrishna
Theoretical Statistics and Mathematics Unit, Indian Statistical Institute ,
India

An *iterative root* of order $n \geq 2$ of a function $f : X \to X$ on a nonempty set X is a function $g : X \to X$ such that $g^n = f$. In this talk we present some new results on the nonexistence of iterative roots of multifunctions on arbitrary sets. In contrast to most known results, our approach allows the presence of arbitrarily many set-value points. The talk is based on recent joint work with Witold Jarczyk.

On Weighted Convex Graphs

Angshuman Robin Goswami
University of Pannonia, Hungary

The primary objective of this presentation is to introduce Krein–Milman-type theorems and Hyers–Ulam-type stability results for weighted convex graphs. Inspired by the concept of sequential convexity, we define four notions of convex graphs. We establish that convex sequences can be embedded into specific graphs so that the defined graph-theoretic convexity properties are preserved. In addition, several structural characteristics, convex minorants, and sandwich-type results will also be discussed. Overall, the presentation proposes a unified framework connecting discrete convex analysis, weighted graph theory, and stability theory, and highlights how classical convexity concepts can be meaningfully transferred to graph-theoretic settings.

When are solution spaces finite-dimensional? Linear differential-algebraic equations revisited

Eszter Gselmann
University of Debrecen, Hungary
(joint work with Zsolt Páles)

We study the solution spaces of homogeneous linear differential-algebraic equations with constant coefficients of the form $E\dot{z} = Az$, where $E, A \in \mathbb{C}^{m\times n}$. Using the Kronecker canonical form of the matrix pair (E, A), we provide a description of solutions.

We show that the behavior of the solution space depends on whether (E, A) is degenerate or nondegenerate. In the nondegenerate case, the solution space is finite-dimensional and consists of exponential polynomials whose spectra are determined by the characteristic polynomial of the pair. Moreover, we give explicit algebraic conditions for the coefficients and prove that the dimension coincides with the degree of this polynomial.

We extend these results to higher-order systems by reduction to first-order equations, obtaining analogous characterizations and finiteness results.

Exact conditions for discrete refinements on both sides of the Hermite-Hadamard inequality

László Horváth
Department of Mathematics, University of Pannonia, Hungary

In this talk, using a recent result on majorization type inequalities, we give necessary and sufficient conditions for discrete refinements of both sides of the classical Hermite-Hadamard inequality. As applications, we describe special discrete refinements of the Hermite-Hadamard inequality in full detail, obtain refinements of Hermite-Hadamard type inequalities for log-convex functions, solve an open problem raised by Simić, and then give new refinements of the Heinz inequality.

Distributed Halpern Iteration: Strong Convergence to a Common Fixed Point

Szymon Ignaciuk
University of Life Sciences in Lublin, Poland
(joint work with Paweł Kluza, Paweł Kurasiński, Zdzisław Otachel, Andrzej Wiśnicki)

A distributed Halpern iteration (D-HI) for finding a common fixed point of a finite family of nonexpansive operators in a real Hilbert space is introduced. Each agent is assumed to know only its local operator and to exchange estimates with neighbouring agents over a fixed, strongly connected directed graph. Numerical experiments in high-dimensional settings indicate a clear advantage of Halpern-type methods over Krasnosel'skiĭ–Mann-type schemes. The observed behaviour suggests that this advantage may be connected with the norm-convergence properties of Halpern iterations, especially in high-dimensional regimes.

On selections of generalized convex set-valued maps

Daniela Ioana Inoan
Technical University of Cluj-Napoca, Romania
(joint work with Dorian Popa)

An interesting question arising about set-valued maps that satisfy a functional inclusion, is the existence of a selection verifying an equation related to that inclusion. This problem comes from the multivalued method used in Ulam stability problems.

We consider a generalized convex set-valued mapping defined on a grupoid with a bisymmetric operation, and obtain selections that satisfy certain functional equations.

As a consequence, follow some investigations of Hyers-Ulam stability for functional equations on bisymmetric grupoids.

Iterates of Möbius transformations and their dynamics

Justyna Jarczyk
Institute of Mathematics, University of Zielona Góra, Poland
(joint work with Steven Finch and Witold Jarczyk)

Progress in the study of fractional iterates of Möbius transformations, that is mappings $f : \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$ of the form $f(z) = \frac{az+b}{cz+d}$, where $a, b, c, d \in \mathbb{C}$, $c \neq 0$, and $ad - bc \neq 0$, is presented. Their form can be obtained using solutions of the Schröder equation. The latter also allow us to find forms of semiflows into which the transformation under study can be embedded.

Recent results in the theory of means – a personal selection

Witold Jarczyk
Institute of Mathematics, University of Zielona Góra, Poland

During the talk several recent results dealing with means will be presented. Unfortunately, not all those worth presenting will be discussed, and only briefly. The speaker is fully responsible for his choice.

N-ary quasi-arithmetic means without regularity

Gergely Kiss
Alfred Renyi Institute of Mathematics and Corvinus University of Budapest, Hungary
(joint work with Ekaterina Shulman)

Quasi-arithmetic means are classically characterized under suitable regularity assumptions, most notably continuity and strict monotonicity. In this talk I discuss a regularity-free version of this theory for symmetric n-variable operations on real intervals. The main result states that if $F\colon I^n \to I$ is reflexive, symmetric, bisymmetric and strictly increasing in each variable, then F is automatically continuous. Consequently, F must be a quasi-arithmetic mean: there exists a continuous strictly monotone generator φ such that

$$F(x_1, \dots, x_n) = \varphi^{-1}\left(\frac{\varphi(x_1) + \cdots + \varphi(x_n)}{n}\right).$$

The proof is constructive. After reducing to compact intervals, we build an order-preserving coding on the set of n-adic rationals and use bisymmetry as a row–column interchange principle to obtain the averaging identity on a dense domain. A dense-domain gap argument then forces continuity. I will also mention how the same method removes the continuity assumption from a Kolmogorov-type theorem for compatible families of means answering a question of Páles.

Strictly monotonic, strictly nonexpansive quasi graph-additive functions

Tibor Kiss
University of Debrecen, Hungary

The talk discusses solutions to the functional equation

$$f(f(-x)+x) = f(-f(x)) + f(x), \qquad x \in \mathbb{R}.$$

We show that the set of solutions is too rich to provide a complete description of all continuous solutions. However, it turns out that strictly increasing, strictly nonexpansive solutions can be characterized to some extent.

References

[1] T. Kiss, *A counterexample to Matkowski's conjecture for quasi graph-additive functions*, Aequationes Math. **100**/2 (2026), Paper No. 27, 8 pp.

[2] J. Matkowski, *Weakly associative functions and means–new examples and open questions*, Aequationes Math. **99**/6 (2025), 2581–2597.

Bernstein–Doetsch theorem for geometric convex functions

Paweł Artur Kluza
University of Life Sciences in Lublin, Poland
(joint work with Mahmood Kamil Shihab)

The celebrated Bernstein-Doetsch Theorem was under consideration of many researchers for last ten decades. In the present paper we give this Theorem in the geometric convexity. The characterization of the geometric convexity is given as well. Finally, we show that the Jensen geometric convex function can be extended to geometric convex.

References

[1] M. K. Shihab, P. A. Kluza, *Bernstein–Doetsch theorem for geometric convex functions*, Journal of Mathematical Inequalities **19**/4 (2025), 1145–1154. doi: 10.7153/jmi-2025-19-73.
[2] M. K. Shihab, P. A. Kluza, *On the h–Additive Functions and Their Symmetry Properties*, Symmetry **17** (2025), https://doi.org/10.3390/sym17020158

Iterative roots of Brouwer homeomorphisms and the four point matching property

Zbigniew Leśniak
Department of Mathematics, University of the National Education Commission, Krakow, Poland

We study orientation-preserving iterative roots of a Brouwer homeomorphism admitting foliations of the plane into invariant leaves. Using an equivalence relation on pairs of invariant leaves induced by the four point matching property, we construct all such roots by extending them iteratively from one maximal parallelizable region to adjacent ones.

References

[1] F. Le Roux, A.G. O'Farrell, M. Roginskaya and I. Short, *Flowability of plane homeomorphisms*, Ann. Inst. Fourier Grenoble **62** (2012), 619–639.
[2] Z. Leśniak, *On limit sets of a Brouwer homeomorphism generating foliations of the plane*, J. Difference Equ. Appl. **31** (2025), 1149–1161.

Approximate convexity and Bernstein–Doetsch type theorems

Judit Makó
University of Miskolc, Hungary

In this talk, an approximate convexity definition will be introduced. Namely, let D be a nonempty, convex, open subset of the normed space X, $\alpha : [0, \infty[\to \mathbb{R}$ be an error function, a function $f : D \to \mathbb{R}$ is called (t, α)-convex, if for all $x, y \in D$,

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) + \alpha(\|x - y\|).$$

I would like to show some properties of (t, α)-convexity and also prove Bernstein–Doetsch type theorems.

On trigonometric φ-Jensen convex functions

Gábor M. Molnár
University of Nyíregyháza, Hungary
(joint work with Zsolt Páles)

In this talk, we propose a trigonometric generalization of φ-Jensen convex functions, extending our recent results from [1]. Specifically, we investigate the functional inequality

$$f\left(\frac{x+y}{2}\right) \le \frac{f(x)+f(y)}{2\cos(p(x-y))} + \varphi\left(\frac{x-y}{2}\right)$$

and discuss the estimation of the error terms concerning rational convex combinations.

References

[1] G. M. Molnár, Zs. Páles, *Estimates for approximately Jensen convex functions*, Acta Math. Hungar. **175**/1 (2025).

On a class of nonlinear operators

Constantin P. Niculescu
University of Craiova, Romania

The aim of my talk (dedicated to the memory of my dear friend and collaborator Sorin G. Gal) is to discuss a class of operators that arose as a natural framework for proving various nonlinear extensions of the Korovkin approximation theorem. See [1], which offers a full account of our joint contribution.

References

[1] S. G. Gal, C. P. Niculescu, *Choquet Integrals and Monotone Sublinear Operators*, Frontiers in Mathematics, Birkhauser, 2026.

On graph-directed conjugacy equations indexed by a two-vertex digraph

Kazuki Okamura
Shizuoka University, Japan

This talk is based on [1, 2]. We introduce graph-directed conjugate functional equations on the unit interval, indexed by the complete two-vertex digraph with self-loops. More precisely, we study solutions (φ_0, φ_1) of

$$g_{i,j} \circ \varphi_j = \varphi_i \circ f_{i,j}, \quad i, j \in \{0, 1\},$$

where the maps $\{f_{i,j}, g_{i,j}\}_{i,j}$ satisfy a compatibility condition. We investigate the singularity and regularity properties of solutions arising from compatible systems of Matkowski–Rus weak contractions.

References

[1] K. Okamura, *Construction of graph-directed invariant sets of weak contractions on semi-metric spaces*, Aequationes Math., **99**/6 (2025), 2631–2656.

[2] K. Okamura, *Singularity for graph-directed conjugate equations indexed by a two-vertex digraph*, arXiv:2603.21536.

Invariance equation for weighted two-variable Bajraktarević means

Zsolt Páles
University of Debrecen, Hungary
(joint work with Amr Zakaria)

Let I be a nonempty open interval. Given two functions $f, g : I \to \mathbb{R}$ such that g is positive and f/g is strictly monotone, the *two-variable weighted Bajraktarević mean* $B_{f,g} : I^2 \times \mathbb{R}_+^2 \to I$ is defined by

$$B_{f,g}(x, y; \alpha, \beta) := \left(\frac{f}{g}\right)^{-1} \left(\frac{\alpha f(x) + \beta f(y)}{\alpha g(x) + \beta g(y)}\right) \qquad (x, y \in I,\ a, b \in \mathbb{R}_+).$$

The pair (f, g) is called the *generator of the mean* $B_{f,g}$. This notion was introduced by M. Bajraktarević in 1958. The aim of this paper is to study the following invariance equation

$$B_{p,q}(B_{f,g}(x, y; t, 1 - t), B_{h,k}(x, y; s, 1 - s); 1, 1) = B_{p,q}(x, y; 1, 1) \qquad (x, y \in I),$$

where $t, s \in (0, 1)$ are fixed constants and $f, g, h, k, p, q : I \to \mathbb{R}$ are considered as unknown functions.

When are local properties of solutions of iterative functional equations actually global?

Paweł Pasteczka
University of Rzeszow, Poland
(joint work with Witold Jarczyk)

We study the extendability of properties of solutions of simultaneous iterative equations. We prove that, under some additional assumptions, if the solution of

$$\varphi(f_t(x)) = g_t(x, \varphi(x)), \quad t \in T, \tag{E}$$

exists, then it is not only unique, but its local properties turn out to be actually global. More precisely, we show that, in the case when the indicated function solves the simultaneous equations, each local property can be propagated to the entire domain.

Systems of the form (E) have been intensively studied in last 70 years, starting from papers by G. de Rham.

Semicanonical Normal Forms and some Applications

Ludwig Reich
Department of Mathematics and Scientific Computing, University of Graz, Austria

We consider formal power series over $\mathbb{C}$ of the form $F(x) = \sigma x + \sum_{n\geq 2} c_n x^n$ where $\sigma = e^{2\pi i\ell}/m$, $\gcd(\ell, m) = 1$ for some integer $m \geq 1$. A semicanonical normal form with respect to σ is of the form

$$N(x) = \sigma x + \sum_{n\geq 1} d_{nm+1} x^{nm+1}.$$

For each $F(x) = \sigma x + \ldots$, there exists some $S(x) = x + \ldots$ such that $S^{-1} \circ F \circ S = N$, a semicanonical nomal form with respect to σ. In general S is not uniquely determined. If F is not linearizable, there exists some n_0 such that $N(x) = \sigma + d_{n_0m+1} x^{n_0m+1} + \ldots$ where $d_{n_0m+1} \neq 0$. Series commuting with N have a structure similar to N. We discuss the existence of semicanonical normal forms and the invariance of n_0 for all series commuting with N.

The robust Orlicz premium principle under uncertainty

Patryk Rela
University of Rzeszów, Poland
(joint work with Jacek Chudziak)

Let $(\Omega, \mathcal{F})$ be a measurable function and $\mu_0 : \mathcal{F} \to [0,1]$ be a capacity. The robust Orlicz premium principle under uncertainty for the risk X is defined through the equation

$$H_{\alpha,S,\Phi}(X) := \inf\left\{k > 0 : \sup_{\mu \in S} E_\mu\left[\Phi\left(\frac{X}{k}\right)\right] \leq 1-\alpha\right\}, \tag{1}$$

where $\alpha \in [0,1)$ is a given parameter, S is a family of capacities μ, the function $\Phi : [0,\infty) \to [0,\infty)$ is a normalized Young function and E_μ is a Choquet integral with respect to μ.
The aim of this talk is to prove the existence of the robust Orlicz premium defined by (1) and to characterize its several important properties.

Roman – The Boss, The Master, The Friend

Maciej Sablik
University of Silesia, Poland
(joint work with Tomasz Szostok)

The talk presents the late Roman Ger.

Nonlinear equivalence of finite-dimensional normed spaces based on approximate Birkhoff–James orthogonality

Rafał Stypka
Department of Mathematics University of the National Education Commission, Krakow, Poland

We consider bijective operators that approximately preserve Birkhoff–James orthogonality in both directions. In this setting, we introduce the notion of an approximate Birkhoff–James isomorphism between normed spaces. Moreover, we define a new class of approximate Birkhoff–James isomorphisms and investigate their influence on the linear-topological structure of normed spaces.

The main result of the presentation shows that if two normed spaces are approximately isomorphic in the sense of Birkhoff–James and one of them is finite-dimensional, then the spaces are isomorphic in the classical sense. Consequently, approximate preservation of Birkhoff–James orthogonality strongly determines the structure of finite-dimensional normed spaces.

Equivalence of Spectral Analysis and Spectral Synthesis

László Székelyhidi
University of Debrecen, Hungary

Spectral synthesis on a locally compact abelian (LCA) group G means that every variety on G admits spectral synthesis; spectral analysis means that every nonzero variety contains an exponential. Recently we characterized those LCA groups on which spectral synthesis holds for all varieties, using a localization method. However, most LCA groups do not have this global property: on a general group, some varieties satisfy spectral synthesis while others fail it. For an individual variety, spectral analysis is a priori much weaker than spectral synthesis. Analysis requires only the existence of exponential functions—common eigenfunctions of all translation operators—in every nonzero subvariety. Synthesis, in contrast, asserts that the exponential monomials generated by these exponentials span a dense subspace in every subvariety. In finite-dimensional linear algebra this distinction disappears: for a commuting family of operators, invariant subspaces always contain eigenvectors, and these eigenvectors (together with their higher-order analogues) completely describe the space. In infinite dimensions this need not hold: there exist varieties with spectral analysis but without spectral synthesis.

The main result of the talk is that such counterexamples are highly exceptional and can be described precisely. In fact, for varieties on $\mathbb{R}^n$ the two notions coincide: spectral analysis already implies spectral synthesis. Combined with our localization framework, this leads to a complete structural understanding of when and how the two notions differ on general LCA groups.

The difference property and higher-order convexity properties of difference operators

Tomasz Szostok
University of Silesia, Poland
(joint work with Teresa Rajba)

We deal with the higher-order version of the problem concerning the properties of the difference operator (see [1],[3]). To this end, we need to generalize Ger's result from [2]. A connection with higher-order Wright convexity is also shown.

References

[1] M. Balcerowski, *On a problem of T. Szostok on functions with monotonic differences*, Aequationes Math. **85**/1–2 (2013), 165–167.
[2] R. Ger, *Convexity and the difference property*, Indag. Math. (N.S.) **24**/4 (2013), 693–699.
[3] T. Rajba, *On functions with monotonic differences*, Ann. Math. Sil. **38**/1 (2024), 93–110.

Generalized inverses of strictly monotone transformations

Péter Tóth
University of Debrecen, Hungary

Let $K \subseteq \mathbb{R}^n$ be a nonempty, convex set. Suppose that the mapping $f : K \longrightarrow \mathbb{R}^n$ is strictly increasing, i. e. it fulfills the inequality

$$\langle\, f(x) - f(y)\,,\; x - y\,\rangle > 0 \qquad \text{for all } x, y \in K, x \neq y.$$

For $n = 1$ this is equivalent to the fact that f is strictly increasing on an interval. In our talk we investigate the existence of a monotone generalized left inverse function for f. The question is whether there exist $f^{(-1)} : \operatorname{conv}(f(K)) \longrightarrow K$ such that $f^{(-1)}(f(x)) = x$ for all $x \in K$, moreover

$$\langle\, f^{(-1)}(u) - f^{(-1)}(v)\,,\; u - v\,\rangle \geq 0 \qquad \text{for all } u, v \in \operatorname{conv}(f(K))\,.$$

We give a positive answer in the case when K is closed. We discuss the higher dimensional setting and outline some applications as well.

Some considerations on the equation $g \circ g = f$ in $C(\mathbb{R})$

Andrei Vernescu
Valahia University, Romania

We recall and give some results concerning the equation $g \circ g = f$ in $C(\mathbb{R})$ and solve this equation in some particular cases.